THERMOELECTRIC LINK FINITE ELEMENT FOR FGM MATERIALS

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1. Introduction and Problem description

The paper deals with two-way coupled thermoelectric analysis of link conductor made of Functionally Graded Material (FGM) using new-derived FEM equations. Accuracy and effectiveness of the new approach compared to classic approach in computer modelling of such systems will be introduced.

Besides Joule heat, thermoelectric effects describe direct conversion of thermal energy into electric energy (Seebeck effect) and conversion of electric energy into the temperature difference within the system (Peltier effect). All these effects are mathematically described by constitutive equations [1]:

$$\begin{aligned} \mathbf{q} &= [\![\Pi]\!] \cdot \mathbf{J} - [\![\lambda]\!] \cdot \nabla T \\ \mathbf{J} &= [\![\sigma]\!] \cdot (\mathbf{E} - [\![\alpha]\!] \cdot \nabla T) \end{aligned} \tag{1}$$

where $\mathbf{q}$ [Wm^{-2}] is heat flux vector, $\mathbf{J}$ [Am^{-2}] is electric current density vector, $[\![\Pi]\!]$ [V] is Peltier coefficient matrix, $[\![\lambda]\!]$ [Wm^{-1}K^{-1}] is thermal conductivity matrix, T [K] is absolute temperature, $\mathbf{E}$ [Vm^{-1}] is electric field intensity vector, $[\![\sigma]\!]$ [Sm^{-1}] is electric conductivity matrix and $[\![\alpha]\!]$ [VK^{-1}] is Seebeck coefficient matrix. These constitutive equations are coupled by set of governing equations for static thermal and electric fields:

$$\begin{aligned} \nabla \cdot \mathbf{q} &= P \\ \nabla \cdot \mathbf{J} &= 0 \end{aligned} \tag{2}$$

where P [Wm^{-3}] is heat generation per volume unit.

Let us consider FGM conductor with length L [m], and rectangular cross-section area A [m^2] (height h [m] and width b [m]) with nodes symbolically denoted “0” and “L”, see Fig. 1.

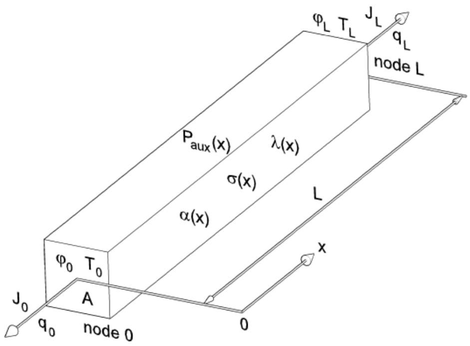

Fig.1: *Two-nodal conductor for thermoelectric analysis.*

In FG materials the material properties are different for every point of the system. This material behaviour can be reduced to the 1D change of material properties (x-direction) using homogenization process, further described in [2]. When we express the differential equations (1) and (2) for 1D system we get the system of differential equations with non-constant coefficients and with right-hand side. This type of equations can be effectively calculated using original method further explained in [3]. The method includes calculation of so-called transfer functions $c(x)$ and $b(x)$. Using this method for solving the system of coupled thermoelectric 1D differential equations we can derive the system of FEM equations for FGM link element that supports thermoelectric effects:

$$\begin{bmatrix} c_0(L)+\frac{\alpha_0 J_0}{\lambda_0}c_1(L) & -1 \\ c_0(L)-\frac{c_1(L)c_0^{'}(L)}{c_1^{'}(L)} & \frac{c_1(L)\alpha_L J_L}{c_1^{'}(L)\lambda_L}-1 \end{bmatrix}\begin{bmatrix} T_0 \\ T_L \end{bmatrix}=\begin{bmatrix} \frac{c_1(L)}{\lambda_0}q_0-\sum_{j=0}^{g}\varepsilon_j b_{j+2}(L) \\ \frac{c_1(L)}{c_1^{'}(L)}\left(\frac{-q_L}{\lambda_L}+\sum_{j=0}^{g}\varepsilon_j b_{j+2}^{'}(L)\right)-\sum_{j=0}^{g}\varepsilon_j b_{j+2}(L) \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} -c_0(L) & 1 \\ -c_0(L)+\frac{c_1(L)c_0^{'}(L)}{c_1^{'}(L)} & 1 \end{bmatrix}\begin{bmatrix} \varphi_0 \\ \varphi_L \end{bmatrix}=\begin{bmatrix} -\frac{c_1(L)}{\sigma_0}J_0-c_1(L)\alpha_0 T_0^{'}+\sum_{j=0}^{g}\varepsilon_j b_{j+2}(L) \\ -\frac{c_1(L)}{c_1^{'}(L)}\left(\frac{-J_L}{\sigma_L}+\alpha_L T_L^{'}+\sum_{j=0}^{g}\varepsilon_j b_{j+2}^{'}(L)\right)+\sum_{j=0}^{g}\varepsilon_j b_{j+2}(L) \end{bmatrix} \quad (4)$$

In [2] there are presented also equations for calculation the primary variables for chosen points within the link element.

2. Numerical experiment

Let us consider electric conductor with rectangular cross-section according to Fig. 1. Its length is $L = 500$ [mm], height $h = 10$ [mm] and width $b = 20$ [mm]. Let the conductor consists of mixture of two component materials – matrix (index m) with constant electric conductivity $\sigma_m(x,y) = 1.429 \times 10^6$ [Sm^{-1}] and thermal conductivity $\lambda_m(x,y) = 2$ [$\mathrm{Wm}^{-1}\mathrm{K}^{-1}$], and fibre (index f) with electric conductivity $\sigma_f(x,y) = 1.111 \times 10^7$ [Sm^{-1}] and thermal conductivity $\lambda_m(x,y) = 400$ [$\mathrm{Wm}^{-1}\mathrm{K}^{-1}$]. Volume fraction of individual components is functionally changed according to chosen polynomial:

$v_f(x,y) = 0.7125 - 7.2214x^2 + 9.6286x^3 + 92.500y - 1658.57x^2y + 2211.43x^3y - 2500y^2 + 1.5514 \times 10^5x^2y^2 - 2.0686 \times 10^5x^3y^2 - 9 \times 10^5y^3 + 1.08 \times 10^7x^2y^3 - 1.44 \times 10^7x^3y^3$ [-]

$v_m(x,y) = 1 - v_f(x,y)$ [-]

Let us consider final Seebeck coefficient for whole conductor according to chosen polynomial function:

$\alpha(x) = -4 \times 10^{-4} + 28 \times 10^{-4}x^2$ [VK^{-1}]

We assume static state for thermoelectric analysis. In nodes 0 and L there are electric potentials and temperatures specified and there is variable auxiliary heat generation in the conductor, so boundary conditions are:

$\varphi(0) = 0.11$ [V]; $T(0) = 273$ [K]

$\varphi(L) = 0$ [V]; $T(L) = 283$ [K]

$P_{aux}(x) = 2 \times 10^5 - 6.4 \times 10^6x^5$ [Wm^{-3}]

We also created 2D model in code ANSYS [4], we used 55 000 PLANE223 elements (8 node quad-elements). The task was also solved in software Mathematica [5], where the differential equations (3) and (4) with specified boundary conditions and homogenized material properties were numerically solved using iterative algorithm. Finally, the task was also solved by only one our new developed two-nodal link element using FEM equations (3) and (4) for nodal points of the link and for chosen points within the link. In Fig. 2 and Fig. 3 we can see calculated longitudinal distribution of the electric potential and temperature in the conductor, respectively. Summary of calculated results is in Tab. 1.

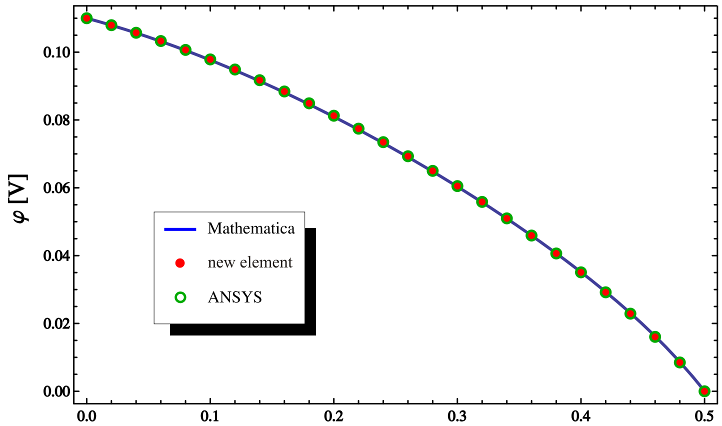

Fig.2: *Distribution of the electric potential through the length of conductor.*

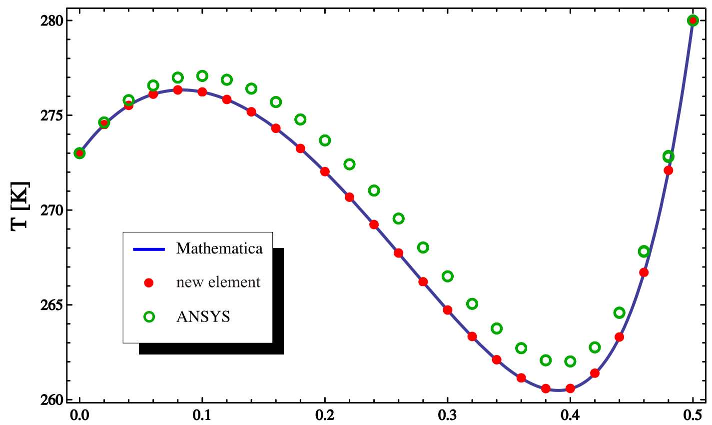

Fig.2: *Distribution of the temperature through the length of conductor.*

Tab. 1. *Comparison of calculated electric and thermal quantities for chosen layers and homogenized values in nodal points of the conductor.*

$J_{\text{layer},\text{node}} \times 10^5$ [Am^{-2}]	$J_{1,0}$	$J_{1,L}$	$J_{6,0}$	$J_{6,L}$	$J_{11,0}$	$J_{11,L}$	J_0^H	J_L^H
new element	6.0532	21.3455	11.0189	8.0888	14.6612	8.3291	10.8288	10.8288
ANSYS	6.1584	21.5903	11.2089	8.1852	14.9130	8.4328	-	-
Mathematica	-	-	-	-	-	-	10.8274	10.8274

$q_{\text{layer},\text{node}} \times 10^4$ [Wm^{-2}]	$q_{1,0}$	$q_{1,L}$	$q_{6,0}$	$q_{6,L}$	$q_{11,0}$	$q_{11,L}$	q_0^H	q_L^H
new element	-7.7619	7.9057	-14.5377	4.6420	-19.5074	4.7011	-14.2730	5.2933
ANSYS	-7.9756	9.2951	-14.9012	4.9339	-19.9653	4.9732	-	-
Mathematica	-	-	-	-	-	-	-14.1887	5.2594

There is small difference in the results (secondary variables) between ANSYS solution and calculation using the new approach in nodal points because of substitutional functions used for conversion non-polynomials into polynomials during iterative process. But we can see from Fig. 2 and Fig. 3 that obtained results correspond to ANSYS 2D simulation very well. Differences in the results for primary variables in the conductor inner region are due to fact that our approach is based on reduction of the real 3D system into 1D problem.

3. Conclusion

New FEM equations with consideration Joule heat, auxiliary heat, and thermoelectric effects, like Seebeck and Peltier effects, were successfully used for demonstration the effectiveness and accuracy of our new approach in calculation of the thermoelectric effects in link conductor. Numerical example with good agreement between calculations with just only one new link element and commercial FEM code that uses numbers of classic elements have been presented. The new approach fully agrees with numerical solution for 1D differential equation of thermal and electric fields calculated using iterative algorithm.

Acknowledgement

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